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PROBLEM SOLVING AS NECESSARY KNOWLEDGE FOR TEACHER TRAINING – MATHEMATICS *FOR* TEACHING[α]

La resolución de problemas como un saber necesario en la formación de profesores, una matemática para *enseñar*

Rosilda dos Santos Morais[β]

Abstract. This paper discusses the themes «problem solving» and «Problem Solving» as topics of different natures. To do so, this research work addresses the question: Has problem solving been used as knowledge to teach «*savoir à enseigner*» and not as knowledge for teaching «*savoir pour enseigner*?». Based on methodological and theoretical references, such a question is the point when Problem Solving, in capital letters, is highlighted and claims the epistemological status of knowledge *for* teaching, mathematics *for* teaching. Moreover, it claims that such knowledge *for* teaching should become the central point of teacher training as regards mathematics and mathematics teaching. To support such claims, we used official Brazilian documents, such as the new *Base Nacional Curricular Comum* (BNCC – National Curriculum Common Base, in free translation), all of which stress the importance of using problem solving. Finally, we discuss the role played by George Polya as a teacher trainer.

Keywords: Mathematics *to* teach; Mathematics *for* teaching; Teacher training; History of mathematics education.

Resumen. *El presente texto problematiza «resolución de problemas» y «Resolución de problemas» como cosas de diferentes naturalezas. Con este fin, guiado por la pregunta: ¿Se ha trabajado la resolución de problemas durante muchos años en términos de conocimiento que se debe enseña*r «savoir à enseigner» *y no como conocimiento para enseñar* «savoir pour enseigner»*? La resolución de problemas gana la escena y se reclama basado*

[α] Prior to this article, an essay was presented in the 4th Ibero-American Congress of History of Mathematics Education (IV-CIHEM), in the «communications» category, in Murcia, Spain, in November 2017. Available at: https://repositorio.ufsc.br/handle/123456789/188824 Accessed on September 9th 2019.

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en marcos teóricos y metodológicos, estado epistemológico de saber enseñar, matemáticas para enseñar. Además, se afirma que tal conocimiento para enseñar está en el corazón de la formación docente en matemáticas y su enseñanza. Para respaldar esta afirmación, destacan los documentos oficiales de Brasil, por ejemplo, la nueva Base Nacional Curricular Común (BNCC), todo lo cual subraya la importancia del trabajo de resolución de problemas. Finalmente, cuestionamos el papel desempeñado por George Polya como formador de maestros.

Palabras-clave: *Matemáticas* a *enseñar; Matemáticas* para *enseñar; Formación de profesores; Historia de la educación matemática.*

ESSAY 1

Using the same expression to say different things, such as «Problem Solving» and «problem solving», «mathematics education» and «Mathematics Education», is supported by Foucault in *The Order of Things*. While discussing the first form of language, when it was given to men by God, he stated that

> [...] it was an absolutely right and transparent sign because it made them similar. Names were assigned to the things they designated, just like strength is written all over a lion's body, like royalty is in the eyes of an eagle, like the influence of the planets is marked on men's foreheads; by way of similarity.[1]

However, «language no longer mimics the *things* that it names immediately [...]».[2] For instance, here is what literature says about «Problem Solving» and «problem solving» or «Mathematics Education» and «mathematics education»[3] —such expressions are written the same, but they mean different *things*, which are not antagonistic, though.

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[1] Michel Foucault, *As palavras e as coisas* (São Paulo: Martins Fontes, 1999), 52.

[2] Foucault, *As palavras e as coisas*, 52.

[3] In this article, «problem solving» is understood as a common practice in mathematics classes and «Problem Solving» means the teaching methodology through which one can learn mathematics — it is a methodology. As far as «mathematics education» and «Mathematics Education» are concerned, according to Valente they mean different «things». The former refers to mathematics teaching and learning processes since time immemorial, thus constituting a research theme for studies related to the history of mathematics education. Anyhow, it is necessary to draw such distinction so that one will not think that «history of mathematics education» means only the studies conducted in and after the 1980s [the Brazilian case], or that it is restricted to the history of the research field». The

In order to set a more specific focus, this paper has the objective of discussing «Problem Solving» and «problem solving» as two different «*things*», but, as stated before, they are not antagonistic expressions.

PRESENT EVIDENCE

When the *Base Nacional Comum Curricular* (BNCC – National Curriculum Common Base, in free translation) approaches the curriculum in the «Field of Mathematics», it highlights that students should be led to ask questions, to identify and investigate problems, to propose and to test hypotheses, to «elaborate arguments and explanations, to plan and to perform experimental activities and field research, to report and communicate conclusions, and to search for solutions to practical problems that involve knowledge of Natural Science».[4] Furthermore, the document stresses that students should

> [...] be encouraged to find different solutions to problems, which will allow them to use their knowledge and different resources to solve a problem, such as drawings, graphs, charts, schemes and various supporting materials. For problem solving, students must be guided, from the beginning, to read and interpret the information contained in it, to build a solution strategy, to apply and to confront the solution found. In a similar way to the literacy process, in the field of language, in the beginning, problems might be enunciated orally, based on situations experienced in the classroom.[5]

Further ahead in the same document, about the elaboration of problems, we read that:

latter expression «designates the recent academic field, an area of investigation into the teaching and learning of mathematics. In Brazil, a founding reference to this field was the establishment of SBEM —Sociedade Brasileira de Educação Matemática (Brazilian Mathematics Education Society, in free translation)— in the year of 1988». Wagner R.Valente, «Oito temas sobre história da educação matemática», *REMATEC* 8, no. 12 (2013): 24.

[4] Available at: http://basenacionalcomum.mec.gov.br/images/BNCC_EI_EF_110518_versaofinal_site.pdf Accessed on 04-09-2019, 137.

[5] BNCC, 272.

> [...] since an early age, children are curious and, when they face a mathematical situation, like in other situations, they must be encouraged to question, to criticize and to investigate. When they are solving a problem that emerges during a game, for instance, in their following turn, they ask the same questions that were asked in their previous turn, which constitutes an example of problem elaboration attitude. Later, when they are facing research results that can even involve measurements, it is reasonable to expect that students who were encouraged to question since an early age will pose interesting questions about the data — about the results derived from such data, and the relations among the data. Such an investigative attitude — elaborating new problems from other problems — helps develop mathematical thinking.[6]

Based on what these extracts show, in the context of the BNCC, «problem solving» is a common practice in mathematics classes through which students will effectively learn mathematics, build their mathematical thinking and become investigative and critical, not only in mathematics classes, but in their lives.

Considering the theoretical assumption that «problem solving» and «Problem Solving» «speak» of different *things*, what does the latter consist of? That is what this paper will discuss —it will approach «Problem Solving» as *something* that encompasses «problem solving», ways of doing mathematics, of teaching and of learning mathematics, ways of «training for life».

From the excerpts of the BNCC, it is possible to identify the importance of problem solving in mathematics classes not only as a purpose in itself, but as a training subject. Nevertheless, approaching such theme is not an easy task because the «art of problem solving» involves matters of, at least, two natures: proper reading and interpretation of the problem. Each one of these specificities encompasses others, for instance, interpreting a problem reflects how students appropriate what the problem asks of them, what questions it contains, what solution strategies are possible, previous acquisition of knowledge that will help students in the problem-solving process, students' engagement in the task of solving

[6] BNCC, 272.

a problem, and so on. The whole process does not happen in an isolated way, in which students work by themselves. On the contrary, students are guided by the teacher, who will gradually develop a problem-solving culture in the classroom.

A question of interest to our study is: How should teachers organize their classes with problem solving in order to develop a problem-solving culture in their classes? What *competency*[7] must they harness to perform this task?

EVIDENCE FROM THE PAST

Agreeing to Machado[8], Veiga-Neto[9] states that «Studying the emergence of an object —concept, practice, idea or value— is to perform a historical analysis of the political conditions of discourse possibility that instituted and "housed" said object». Veiga Neto complements this quote affirming that «the point is not where it came from, but how and when it emerged». In the question presented in the previous topic, it was possible to observe that the teacher plays the role of developer of a problem-solving culture in the classroom. Moreover, according to the BNCC, problem solving plays an important part in students' training.

When we take all of these aspects into consideration, we understand that «a historical analysis of the political conditions of discourse possibility that instituted and housed said object» means studying the emergence of said object —problem solving is not an end in itself, but *knowledge for teaching*, *mathematics for teaching*.[10] There are two issues to be approached here: problem solving as an element to provide training to individuals —students— and problem solving as necessary *knowledge* to

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[7] Bernard Rey defines *competency* as the capacity an individual has to accomplish a task, in «Les compétences professionnelles et le curriculum: des réalités conciliables?», in *Savoirs Professionnels et curriculum de formation*, orgs. Yves Lenoir and Marie-Hélène Bouillier-Dutot (Canadá: Les Presses de l'Université Laval, 2006), 94.

[8] Roberto Machado, «Por uma genealogia do poder», cited by Alfredo Veiga-Neto in *Foucault & a Educação* (Belo Horizonte: Autêntica, 2003), 61.

[9] Veiga-Neto, *Foucault & a Educação*, 61.

[10] Wagner R. Valente, Luciane F. Bertini, Neuza B. Pinto and Rosilda S. Morais, *A matemática na formação de professores e no ensino: processos e dinâmicas de produção de um saber profissional, 1890-1990* (São Paulo. Projeto Temático, Processo n. 15751-2, FAPESP, 2017).

train teachers, whose expertise should train individuals, as stated by the BNCC. This topic analyzes specifically the second case, that is, problem solving as necessary *knowledge* for teaching training —a working tool for training.

It is possible to observe systematic studies on problem solving since the 20th century, which can be justified by the fact that, at that time, the social context involved industrialization, urbanization and immigration, all of which would indicate different conceptions about what knowledge would be more suitable to be taught in schools. With that background, some researchers discussed problem solving: some of them more diligently, others less industriously, but each group according to their own interests.

In the first half of the 20th century, Edward Lee Thorndike discussed the role that mathematical problems could play in students' training, with emphasis on the types of problems that should be used. For him, more than discussing the importance of paying attention to problem solving, it was necessary to consider the types of problems. In this sense, in an attempt to call into question psychological theories in effect at that time —Mental Discipline Theory (MDT), for instance— Thorndike wanted to make the theory that he advocated, Connectionism, circulate. It assumed that learning consists of addition, elimination and organization of connections.[11]

With the publication of *The New Methods in Arithmetic*, in 1921, Thorndike highlighted the teaching of arithmetic as a form of help for life, so mathematical problems should guide students in that direction. One chapter of the book focused on problem solving. However, it is necessary to stress that he emphasized the types of problems as possibilities of applying his connectionist theory because, according to him, problems —as they were being taught— would not prepare students for the reality that they faced.[12]

[11] Rosilda dos Santos Morais, «O processo constitutivo da Resolução de Problemas como temática de pesquisa em Educação Matemática — um inventário a partir de documentos dos ICMEs» (Doctoral Thesis, São Paulo State University, UNESP - «Júlio de Mesquita Filho» University, 2015).

[12] Edouard Lee Thorndike, *The new methods in Arithmetic* (Openlibrary.org, 1921). (Acessado em 30-12-2016). URL: http://archive.org/stream/newmethodsinari00thorgoog#page/n136/mode/2up

At that same time, other researchers discussed problem solving, like William Brownell and George Polya. Brownell, in line with Thorndike, discussed problem solving in order to circulate the psychological theory that he advocated: the theory of meaningful learning[13]. By the way, he acted by criticizing the connectionist theory, stating that meaningful learning should gain prominence. The limitation of this text does not allow us to advance into this theme, which was broadly discussed by this author in 2015.[14]

George Polya has been extensively mentioned, especially when it comes to problem solving. Several participants in the community that investigates the theme say that Polya was «the father of problem solving». A justification for the use of such expression was given by Guimarães[15] who stressed: «George Polya was the only mathematician to combine, throughout his distinct career, a deep investigation into a broad theme with an ever-present interest in the teaching of mathematics».[16]

Considering what has been said, we notice that, besides discussing problem solving, Polya was regarded as a renowned mathematician and, with respect to such formation, he taught about problem solving and taught, above all, mathematics *for* problem solving. In other words, Polya was an expert in the field and his condition attributed a different status to problem solving —a status that had not been considered by others that had discussed it before him, that is, the status of *knowledge* for teaching, of mathematics *for* teaching.[17]

Although much has been said about Polya's role in problem solving, this text has, among others, the objective of highlighting his role in the training of educators who teach mathematics. So far, according to what has been researched by the author of this text, very little has been discussed about it.

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[13] William Brownell, «The progressive nature of learning in mathematics», *Mathematics Teacher. 100 Years of Mathematics Teacher*, NCTM, vol.100, Special Issue (Reston, VA: NCTM, 2006): 26-35. First edition 1944.

[14] Morais, *O processo constitutivo da Resolução de Problemas*, 2015.

[15] H. M. Guimarães, *Polya e as Capacidades Matemáticas*, *Educação e Matemática* 114 (2011): 28-36.

[16] Guimarães, *Polya e as Capacidades Matemáticas*, 29.

[17] Valente, Bertini, Pinto e Morais, *A matemática na formação de professores e no ensino,* 2017.

THE INSTITUTIONALIZATION OF PROBLEM SOLVING AS KNOWLEDGE *FOR* TEACHING - MATHEMATICS *FOR* TEACHING

The article *Developing Understanding in Mathematics via Problem Solving* by Schroeder and Lester is well-known among the members of the community that investigates problem solving. In the 1980s, a mantra repeated worldwide preached that it was necessary to «make problem solving become the focus of school mathematics». On the theme, Schroeder and Lester say that there have been distinct interpretations of the meaning of that call and, for this reason, it was divided into three problem solving teaching approaches: «teaching *about* problem solving; teaching *for* problem solving; and teaching *via* problem solving».[18]

For the purposes of this text, it is interesting to discuss the first of the approaches, «teaching *about* problem solving». The two others have been extensively discussed in other investigative works, for instance, by Morais and Onuchic[19] and by Allevato and Onuchic.[20] According to Schroeder and Lester, teaching through such approach refers to the method proposed by Polya[21], or to some small variations on it. They stated that the model describes a group of four independent stages in the problem-solving process of mathematical problems: (1) understanding the problem; (2) elaborating a plan; (3) carrying out the plan; and (4) performing backtracking.

Students are explicitly taught to think about the stages, which, according to Polya, are the same as the ones that good problem solvers use when they solve mathematical problems —they are encouraged to

[18] Thomas L. Schroeder an d Franklin K. Lester Jr., «Developing Understanding in Mathematics via Problem Solving», in *New Directions for Elementary School Mathematics*, orgs. P. R. Trafton and A. P. Shulte (Reston: VA: NCTM, 1989), 32.

[19] Rosilda dos Santos Morais and Lourdes de la Rosa Onuchic, «Uma abordagem histórica da Resolução de Problemas», em *Resolução de problemas — Teoria e Prática*, orgs. Lourdes de la Rosa Onuchic, Norma Suely G. Allevato and Fabiane Cristina H. Nogutti (Jundiaí: Paco Editorial, 2014), 17-34.

[20] Norma Suely Gomes Allevato and Lourdes de la Rosa Onuhic, «Ensino-aprendizagem-avaliação de Matemática: porque através da Resolução de Problemas?», in *Resolução de problemas — Teoria e Prática*, orgs. Lourdes de la Rosa Onuchic, Norma Suely G. Allevato, Fabiane Cristina H. Nogutti and Andresa Maria Justulin (Jundiaí: Paco Editorial, 2014), 35-52.

[21] George Polya, *A arte de resolver problemas* (Rio de Janeiro: Interciência, 1995).

become aware of their own progress through the stages when they are solving problems by themselves. Furthermore, they learn a number of strategies or heuristics that they might choose to use to carry out their plan. Some of the strategies involve looking for patterns, simple problem solving and working with backtracking. In addition, teaching *about* problem solving includes experiences with solving real problems, but it always involves a great deal of explicit discussion and the teaching of how the problems are solved.[22]

Based on what has been discussed so far, it is necessary that there is someone teaching about problem solving. The word «teaching» is related to «intentional learning»,[23] in which there is essentially a learner and a teacher or educator, such as the historical development «of teaching institutions and training initiatives, which results in an ever-bigger number of individuals who have the primary task of training other people».[24] This is the teacher's task.

Literature has highlighted the four stages for problem solving as identified by Polya. It is important to stress, however, that it has produced a reductionist effect to the broad research done by that renowned researcher and professor in such a way that it is reasonable to state that —at least in a discursive scope—, when it comes to discussing Polya, his research is routinely related to the four stages for problem solving.

In an attempt to break such reductionism, this paper aims at discussing the role played by George Polya in the training of mathematics teachers, which is the theme in the next topic.

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[22] Schoreder and Lester, *Developing Understanding*.

[23] Hofstetter and Schneuwly state in «Saberes: um tema central para as profissões do ensino e da formação» that «intentional learning» is «learning which has as its primary objective to transform the subject itself in contrast to a productive activity», in *Saberes em (trans)formação: um tema central da formação de professores* (São Paulo: Editora Livraria da Física, 2017), 117. Based on studies by Samurçay and Rabardel, in «Modèles pour l'analyse de l'activité et decompétences: propositions», Hofstetter and Schneuwly differentiate both activities by understanding that «when acting, a subject transforms the real (material, social, symbolic); This is what characterizes productive activity. But transforming the real, the subject transforms itself: it is the constructive activity», in «Saberes: um tema central», 117.

[24] Hofstetter and Schneuwly, «Saberes: um tema central», 115.

KNOWLEDGE *TO* TEACH AND KNOWLEDGE *FOR* TEACHING

Teaching *about* problem solving demands, first and foremost, apprehending such knowledge. This text does not discuss problem solving as a teaching resource (emphasizing the BNCC, according to the excerpts presented in the beginning of this article). Our choice has the sole goal of discussing it as *knowledge* — as mathematics *for* teaching.[25]

Hofstetter and Schneuwly advocate that knowledge is a set of objects and instruments in the work of training and teaching. They stress that «knowledge is central in teaching and training institutions and, as a consequence, in the attributions ascribed to the professionals that work there».[26] Supported by Babier,[27] they characterize knowledge in two forms: «the field of "incorporated knowledge", which is inscribed in the semantic zone of capacities, knowledge, competencies, aptitudes, attitudes, professionality»[28] and

> the field of «objectified knowledge», which encompasses realities with the status of representations […] giving way to propositional statements and being the object of a social appreciation sustained by a transmission-communication activity. They — the representations — consequently have a distinct existence from those who enunciate them or those who appropriate them. They are conservable, cumulative, appropriable.[29]

Following the same line of thinking, Hofstetter and Schneuwly state that their interest is focused on formalized knowledge, trying to conceptualize their role in teaching and training professions. Thus, while considering the characteristics of institutions, they define two constitutive types of knowledge related to the two professions: «knowledge *to* teach, that is, the knowledge that is his or her [the teacher's] object of work;

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[25] Luciane de Fátima Bertini, Rosilda dos Santos Morais and Wagner Rodrigues Valente, *A matemática a ensinar e a matemática para ensinar: novos estudos sobre a formação de professores* (São Paulo: Livraria da Física, 2017).

[26] Hofstetter and Schneuwly, «Saberes: um tema central», 131.

[27] Jean-Marie Barbier, «Introduction», in *Savoirs théoriques et savoirs d'action*, dir. Jean-Mari Barbier (Paris: Press Universitaires de France, 1996), 9, cited by Hofstetter e Schneuwly in «Saberes: um tema central», 131.

[28] Hofstetter and Schneuwly, «Saberes: um tema central», 13.

[29] Hofstetter and Schneuwly, «Saberes: um tema central, 131.

and knowledge *for* teaching, the knowledge that is the teacher's working tools».[30]

If «teaching and training» necessarily have knowledge as their *objects* in such a broad sense, so «the choice of knowledge and its transformation into knowledge *to* teach is the result of complex processes that fundamentally transform knowledge in order to make them teachable».[31] About this point, a question we feel the need to ask is: has problem solving —for many years— been used as knowledge *to* teach and not as knowledge *for* teaching?

In the scope of knowledge *to* teach —a teacher's object of work—, to solve a problem, it was (and still is) enough to deploy concepts and contents that, when applied, led (lead) to the solution. And why has solving a mathematical problem historically been many students «Achilles' heel»?

It seems that the problem precedes the student, i.e. the leaner, and turns to the knowledge that constitutes training —the formation of teacher trainers. Therefore, there is the hypothesis that possessing knowledge *for* teaching, more specifically the knowledge that should constitute the basis of teachers' training, might answer that question. Nevertheless, the configuration of training courses —this text is specifically interested in the formation of educators who teach mathematics— has presented, throughout history, on one hand, content disciplines (Real Analysis, Algebra, Calculus…), and on the other hand, pedagogical disciplines (Didactics, Teaching Practice, Supervised Practice…).

Addressing problem solving as a teaching methodology, to which this text attributes the status of knowledge *for* teaching, is, according to Polya, an attempt to blend the knowledge that constitutes the professional field —the knowledge in which the reference is always the *professional expertise* that encompasses knowledge *for* teaching— with the knowledge emerging from content disciplines, that is, the knowledge related to knowledge *to* teach.

[30] Hofstetter and Schneuwly, «Saberes: um tema central», 131.

[31] Hofstetter and Schneuwly, «Saberes: um tema central», 133.

When Hofstetter and Schneuwly approach teacher trainers, they highlight that they

> train others by teaching knowledge; thus, their function is constitutively defined by knowledge to which they train, or knowledge *to* teach [...]. Such knowledge constitutes an essential object to their work. The contract that connects the professionals with the institutions that employ them defines what they have to teach, which is specified by plans or curricula, by manuals, training devices, prescriptive texts of different types.[32]

The authors also say that «this process may even lead to the creation of knowledge that is typical of educational institutions and necessary to them so that they can perform their function» and that «teacher trainers act through simulating reality and its explanation, elementation and demonstration while taking knowledge into consideration —thus, assuming models of knowledge *to* teach».[33]

Finally, they conclude that teacher trainers' professional activity consists of teaching knowledge in a broad sense while «considering the support of knowledge, of communicable and socially recognized wordings or, in other words, getting help from pedagogical knowledge».[34] Therefore,

> training, like any other human activity, implies the deployment of knowledge for its implementation, for performing this task, this specific craft. And such knowledge constitutes working tools — in this case, *knowledge for training* or *knowledge for teaching* [...] It is mainly knowledge of «the object» of the teaching and training work (of the knowledge *to* teach and of the adult students, their knowledge, their development, their ways of learning, etc.), of teaching practices (methods, procedures, devices, choice of knowledge *to* teach, models of organization and management) and of the institution that defines their field of professional activity (study plans, instructions, purposes, management

[32] Hofstetter and Schneuwly, «Saberes: um tema central», 132.

[33] Hofstetter and Schneuwly, «Saberes: um tema central», 133.

[34] Hofstetter and Schneuwly, «Saberes: um tema central», 133.

> and political structures, etc.). Like in any other profession, all of this knowledge is multiform.[35]

About such multiplicity, Hofstetter and Schneuwly stress that the most influent research on it was done by Schulman. It encompasses seven categories, which are:

> knowledge of subject contents, pedagogical knowledge, knowledge of the curriculum, knowledge about students, knowledge related to educational contexts, knowledge of your own educational purposes and, above all, knowledge that is typical of the profession: «*pedagogical content knowledge, that special amalgam of content and pedagogy*».[36]

Supported by Baumert and Kunert [37], they also highlight that Shulman's typology can be reduced to «a triadic scheme that would be a consensus nowadays: *fachwissenschaftliches, fachdidaktiktisches und pädagogisches Wissen* [knowledge of the subject sciences, knowledge of subject didactics, pedagogical knowledge]».[38]

ADVOCATING FOR PROBLEM SOLVING AS A MATH *FOR* TEACHING

In the beginning of this text, we cited Michael Foucault in *The Order of Things* and highlighted that «language no longer mimics the *things* that it names immediately [...]» and that we would discuss expressions that are written the same —«problem solving» and «Problem Solving»—, but mean different *things*, which are not antagonistic, though.

The title of this topic contains the expression written in capital letters, which is not coincidental. As mentioned before, we attribute here the epistemological *status* of knowledge *for* teaching to problem solving. The complexity of the expression can justify such attribution.

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[35] Hofstetter and Schneuwly, «Saberes: um tema central», 134.

[36] B. Shulman, «Les outils de l'enseignant —Um essai didactique», *Repères* 22 (2000): 19-38, cited by Hofstetter and Schneuwly, «Saberes: um tema central», 134-135.

[37] Baumert, J. and Kunter, M. «Stichwort: Professionelle Kompetenz Von Lehrkräften», *Zeitschrift für Erziehungswissenschaft* (2006): 9, cited by Hofstetter e Schneuwly in «Saberes: um tema central», 134-135.

[38] Hofstetter and Schneuwly, «Saberes: um tema central», 135.

The expression «problem solving» is polysemic. Polya, for instance, did not discuss problem solving as a mere practice, but problem solving as something that was beyond just a technique. Considering what has been discussed so far, beyond practice and technique, since Polya, Problem Solving has been professional knowledge possessed and shared by educators who teach mathematics. The choice to write it in capital letters is justified in order to differentiate it from the technique.

In other words, considering problem solving as professional knowledge situates it in teacher training; it is institutionalized as knowledge *for* teaching, as mathematics *for* teaching. Polya said that an average teacher could not possibly conduct advanced research if he or she did not have the conditions to do so, because solving an unusual mathematical problem demands serious and creative work. Moreover, the development of such creativity should happen in teacher training, with a systematic study, not acquired through mere memorization.[39] What did Polya mean by «creativity»?

Based on the references harnessed so far, the author of this text understands that when Polya proposed working with problem solving in teacher training, he questioned the knowledge that was in effect at that time in that stage of training. As an expert, for reasons that have already been presented in this text, Polya had such professional expertise that he was able to reflect scientifically and «autonomously», which is in agreement with Hofstetter and Schneuwly. They assess that such attitude is positive, for it can stop «creeping deintellectualization» and the process of commercialization of knowledge, which are so common in the society of knowledge.

The theoretical framework harnessed here allowed the author of this paper to understand that Polya advocated that problem solving should ascend from knowledge *to* teach (commercialization of knowledge, problem solving as a stepping stone to spread psychological theories, for example) to the status of constitutive knowledge of teacher training, knowledge *for* teaching, mathematics *for* teaching. Finally, the request for the presence of problem solving in the curriculum of educators who

[39] George Polya, *Mathematical Discovery — on understanding, learning, and teaching problem solving* (United States of America: Library of Congress Catalog Card Number, 1962).

teach mathematics legitimizes its institutionalization as teachers' professional knowledge.[40] Albeit in the final part of this text, we highlight that problem solving as mathematics *for* teaching and problem solving as mathematics *to* teach are interwoven, but possessing mathematics *for* teaching is the professional expertise that characterizes the profession of educators who teach mathematics.

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